

# Dark Matter Halos with Cores from Hierarchical Structure Formation

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We show that dark matter emerging from late decays ( $z \lesssim 1000$ ) produces a linear power spectrum identical to that of Cold Dark Matter (CDM) on all observationally relevant scales ( $\gtrsim 0.1$  Mpc), and simultaneously generates observable constant-density cores in small dark matter halos. We refer to this class of models as *meta*-Cold Dark Matter (*m*CDM), because it is born with non-relativistic velocities from the decays of cold thermal relics. The constant-density cores are a result of the low phase-space density of *m*CDM at birth. Warm dark matter cannot produce similar size phase-space limited cores without saturating the Ly $\alpha$  power spectrum bounds. Dark matter dominated galaxy rotation curves and stellar velocity dispersion profiles may provide the best means to discriminate between *m*CDM and CDM. *m*CDM candidates are motivated by the particle spectrum of supersymmetric and extra dimensional extensions to the standard model of particle physics.

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## INTRODUCTION

In the standard cosmological model, the bulk of the matter in the universe is in the form of Cold Dark Matter (CDM). Structure in CDM is built up hierarchically, from initial density perturbations that are adiabatic and nearly scale-invariant [1]. CDM provides a compelling description of the universe on large scales, but it has not conclusively passed all observational tests on small scales. Two well-known shortcomings are that simulations predict Milky Way-sized galaxies should have nearly an order of magnitude more dark matter substructures than observed satellites [2], and density profiles of low-mass galaxies tend to be less cuspy than those seen in simulations [3, 4, 5, 6] and are often well-fit by constant-density cores. While the former can be alleviated both by astrophysical [7] and cosmological [8, 9] methods, the latter, if ultimately true, may require a more drastic modification of the CDM paradigm [10].

Warm dark matter (WDM) provides an alternative that may alleviate the problems on small scales [11, 12]. Indeed, we *expect* constant density cores in the centers of WDM halos because the initial phase-space maximum can never be exceeded [13]. An additional consequence of the large WDM particle velocities is that its free-streaming length is large. This suppresses the power spectrum on small scales. One problem with WDM as an alternative to CDM, however, is that models with velocities large enough to produce observationally-relevant cores are in conflict with measurement of the Lyman- $\alpha$  forest power spectrum [14]. Indeed, Lyman- $\alpha$  forest analyses suggest that the linear density fluctuation spectrum is nearly identical to the CDM prediction on scales larger than  $\sim 1$  Mpc [15, 16, 17, 18, 19].

It is thus interesting to consider if any modification to CDM can reproduce the pattern of hierarchical structure formation, and also create large phase-space limited cores

in the halos of low-mass galaxies. In this paper we investigate such a dark matter model, which we refer to as “*meta*-Cold Dark Matter” (*m*CDM). We define *m*CDM as particles that emerge relatively late in cosmic time ( $z \lesssim 1000$ ) and are born non-relativistic from the decays of cold particles. As we will show, the *m*CDM model provides both large phase-space cores *and* CDM-like power spectra on  $\gtrsim 0.1$  Mpc scales, consistent with the most stringent measurements of the Ly $\alpha$  forest power spectrum [18, 19]. Specifically, *m*CDM is born late enough that the free-streaming scale is small, but the velocity dispersion is large enough to give rise to a reduced phase-space density. Even though *m*CDM velocities are non-relativistic, the redshifted velocities today are larger than the corresponding velocities for WDM. Previous studies of similar models considered decays with shorter lifetimes  $\sim M_{pl}^2/M_{weak}^3$  (of order a month) to *relativistic* daughter particles [20, 21]. In these models the relationship between the phase-space density and cut-off scale in the power spectrum is similar to that of WDM.

Dark matter in the *m*CDM class arises in many extensions to the standard model of particle physics. For example in supersymmetric models we can consider a sneutrino decaying into a neutrino and gravitino with a lifetime  $\tau \simeq 3.6 \times 10^8$  s ( $100 \text{ GeV}/\Delta m)^4 m_{\tilde{G}}/\text{TeV}$  [22, 23], where  $\Delta m$  is the mass difference between the sneutrino and gravitino, and  $m_{\tilde{G}}$  is the gravitino mass. A similar relation holds for Kaluza-Klein WIMPs decaying to gravitons in theories with extra dimensions. For example, we can consider a 10 TeV gravitino produced from sneutrino decays with a lifetime  $\tau = 5 \times 10^{12}$  s, where the velocity imparted to the gravitino is  $v/c \sim 10^{-3}$ .

## DECAY KINEMATICS

We assume that the decays take the form  $\text{CDM} \rightarrow m\text{CDM} + \ell$ , where the decaying CDM is non-relativistic,  $m\text{CDM}$  is the dark matter today, and  $\ell$  is a light neutral particle. After the epoch of decay,  $a_d$ , the velocity of the daughter particle will be  $v \simeq (p_{cm}/m)(a_d/a)$ , where  $m$  is the  $m\text{CDM}$  mass and  $p_{cm}$  is the center of mass momentum of the daughter particles. We define the average primordial phase-space density for the  $m\text{CDM}$  as  $Q_p \equiv \bar{\rho}/\bar{\sigma}^3$ , where  $\bar{\rho}$  is the mean density and  $\bar{\sigma}$  is the velocity dispersion. Integrating over the phase-space distribution function (presented below) we find [20]

$$Q_p = 10^{-6} \alpha [10^3 p_{cm}/m_{cdm}]^{-3} [10^3 a_d]^{-3}, \quad (1)$$

where  $\alpha = 1.0(0.8)$  for decays in the radiation (matter)-dominated era. Here, and for the rest of the discussion, the units of  $Q$  are  $\text{M}_\odot \text{pc}^{-3} (\text{km s}^{-1})^{-3}$ . Note that for the typical cases we consider,  $(m_{cdm} - m)/m_{cdm} \simeq p_{cm}/m_{cdm} \simeq 10^{-3}$ .

In the case of late decays, the power spectrum is governed not only by  $Q_p$  but also by the epoch of decay,  $a_d$ . We can understand this heuristically in terms of standard free-streaming arguments. In this approximation, power on a comoving scale smaller than a streaming scale,

$$\lambda_{FS} \simeq \int_\tau^{t_0} dt v(a)/a \equiv (p_{cm} a_d/m) \int_{a_d}^1 da a^{-3}/H(a), \quad (2)$$

will be erased. Here  $\tau = t(a_d)$  is the decay lifetime,  $t_0$  is the current epoch, and we have assumed that the decay is non-relativistic. If the decay were to occur well before matter-radiation equality,  $a_d < a_{eq}$ , we would have  $\lambda_{FS} \simeq p_{cm} a_d/m \ln(a_{eq}/a_d) a_{eq}^{1/2} 0.5 H_0^{-1}$ , and the filtering would be driven primarily by the phase-space density variable  $p_{cm} a_d/m$ , and only logarithmically on the decay lifetime through  $a_d$ . If we consider decays in the matter-dominated regime, then the power spectrum filtering scale depends on *both* the decay epoch and the phase-space density:  $\lambda_{FS} \simeq (p_{cm} a_d/m) a_d^{-1/2} 0.5 H_0^{-1}$  ( $\simeq 0.05 \text{ h}^{-1} \text{Mpc}$ ). Thus for long lifetimes ( $a_d \gtrsim a_{eq}$ ), the free-streaming scale is suppressed even if  $Q_p$  is relatively small. For the models we consider, the universe does not re-enter a radiation dominated phase due to the decay.

## POWER SPECTRUM

In order to track the evolution of fluctuations we will need the  $m\text{CDM}$  phase-space distribution. Following [20, 24], we find that

$$f(a, q) = \frac{2\pi^2 \rho(a) \exp(-t_q/\tau)}{q^2 p_{cm} \tau m} \left( \frac{1}{a_q H(a_q)} \right). \quad (3)$$

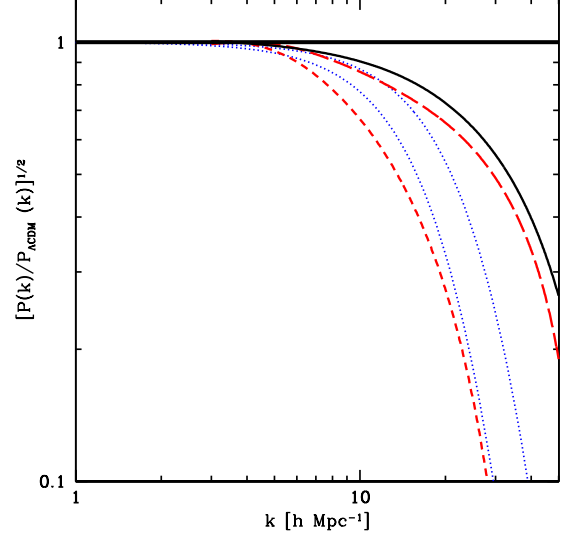


FIG. 1: The ratio of the power spectrum to that of standard  $\Lambda\text{CDM}$  for a variety of different models. The red curves show  $\Lambda+m\text{CDM}$  models with  $\tau = 5 \times 10^{12} \text{ s}$  and  $Q_p = 10^{-5}$  (long-dash) and  $Q_p = 10^{-6}$  (short-dash). The solid black curve is a  $\Lambda+m\text{CDM}$  model with  $\tau = 10^{14} \text{ s}$  and  $Q_p = 10^{-6}$ . Also shown are limits on the warm dark matter power spectrum from the analyses of [18] (dotted, right) and [19] (dotted, left).

The variables  $a_q \equiv q/p_{cm}$ , and  $t_q \equiv t(a_q)$  are characteristic epochs and times for each value of the comoving momentum  $q$ . In the limit of small  $p_{cm}/m$ , we can write all of the perturbations to the phase-space distribution in terms of the density and velocity perturbations. For the decaying CDM, the evolution of these quantities is identical to the standard non-decaying case [25]. This results from the fact that the decays act to remove the same number of particles from every region of space.

To compute the power spectrum we use the full hierarchy of equations governing the perturbation of the phase-space distribution. We use the concordance cosmology [26], and assume that the universe is made flat by a cosmological constant ( $\Lambda$ ), and that the dark matter is in the form of  $m\text{CDM}$ . We can gain insight into the effect of the decays by applying energy conservation and considering a simpler set of equations in the limits where the  $m\text{CDM}$  equation of state and sound speed is negligible. For large scales,  $k \rightarrow 0$ , we have  $y' \equiv \delta'' - \delta''_{cdm} = -y/\tau(1 - 2m_0^2 \rho_{cdm}/m_{cdm}^2 \rho)$ , where  $\delta$  is the  $m\text{CDM}$  density perturbation and the derivatives here are with respect to cosmic time, related to the conformal time,  $\eta$ , by  $d\eta = dt/a$ . We have defined  $m_0^2 = m_{cdm}^2 - m^2$ . For small  $a$ ,  $y$  is driven to zero as  $\eta^2$  and is exponentially damped for  $a > a_d$ . Thus the  $m\text{CDM}$  perturbation ‘catches up’ with the CDM perturbation almost immediately, and the power spectrum retains the form of standard  $\Lambda\text{CDM}$  on scales larger than the horizon at decay.

We expect departures from a  $\Lambda$ CDM power spectrum approximately when  $k^2 c_s^2 \eta^2 \sim 1$ , where  $c_s^2$  is the  $m$ CDM sound speed. For decays in the radiation-dominated regime, this damping scale is given by  $p_{cm} a_d / m$ , and for decays in the matter-dominated regime the scaling is  $(p_{cm} a_d / m) \tau^{-1/3}$ . Thus the damping scale for matter-dominated decays is set by  $Q_p$  and  $\tau$ , while for decays during the radiation era the scale depends only on  $Q_p$ . As one would expect, these are the same scalings we obtained earlier with our simple free-streaming analysis.

In Figure 1 we show the ratio of the power spectrum for a variety of  $\Lambda + m$ CDM models to  $\Lambda$ CDM. The dotted lines are warm dark matter models that correspond to the bounds from recent analyses of the Lyman- $\alpha$  forest [18, 19]. The model with  $\tau = 5 \times 10^{12}$  s and  $Q_p = 10^{-6}$  is potentially inconsistent, though these bounds can clearly be evaded with a  $Q_p = 10^{-5}$  model. An example model with a longer lifetime of  $10^{14}$  s and  $Q_p = 10^{-6}$ , however, is clearly consistent with both bounds, which is what we predict from the above scalings. Further detailed modeling of the Lyman- $\alpha$  power spectrum on small scales thus provides an excellent probe of viable  $m$ CDM models.

## PHASE-SPACE CORES

We can compare the primordial phase-space values  $Q_p$  to the phase-space densities deduced from measurements of galaxy rotation curves. Among the highest-resolution rotation curves for low-mass spirals are those presented by [3] who use two-dimensional H $\alpha$  and CO velocity fields. When the dark matter halo components of these galaxies are fit to cored density distributions, the average central phase-space densities are  $2 \times 10^{-6}$  [3, 14].  $Q_p$  cannot be smaller than this value.

In order to connect  $Q_p$  to the maximum phase space density in a halo,  $Q_0$ , we must consider halo formation. We apply a general argument for collisionless collapse that is much stronger than the Tremaine-Gunn bound for distributions with maximum fine-grained phase space density much larger  $Q_p$ . Specifically, the Excess Mass Function (EMF), defined in [27], must always decrease. Following the approach of [20], we calculate the EMF of primordial  $m$ CDM using Eq. 3 and demand that its value for all phase-space densities be greater than the EMF of the dark matter halo profile. This procedure gives us the maximum central phase space density of the dark matter halo profile that is consistent with a given primordial momentum distribution.

The procedure depends on the details of the decay process as well as the assumed dark matter halo profile. We will restrict our attention here to the Milky Way satellite galaxies, and this motivates a general parameterization for the dark matter halo [28] that interpolates between a truncated NFW profile and a profile with a core –  $\rho(r) \sim \exp(-r/b)/(c+r)/(r+a)^2$ . For parameters

that are broadly consistent with the Milky Way dwarf-satellite galaxies we find numerically that  $Q_0 \lesssim 30Q_p$  for  $Q_p = 10^{-5}$  and  $Q_0 \lesssim 50Q_p$  for  $Q_p = 10^{-6}$ .

The above procedure also provides an estimate of the *minimum* core size. We find that the minimum value of  $c$  ranges between 300 to 800 pc for  $Q_p = 10^{-6}$  depending on the value of  $a$  (restricted to be greater than  $c$ ) for a halo with mass within 1 kpc of  $10^7 M_\odot$  (which results in a central stellar velocity dispersion  $\sim 10$  km/s for a typical satellite dwarf galaxy). The lifetime is consistent with the model in Figure 1. A decrease in halo mass to  $10^6 M_\odot$  increases minimum  $c$  range to 1.5–2.5 kpc, while increasing mass to  $10^8 M_\odot$  decreases it to 150–300 pc range. The minimum  $c$  scales approximately as  $1/\sqrt{Q_p}$  in this range of parameter space.

We caution that the above exercise only provides us with the minimum possible core size. The merging process is expected to increase this core size [28] and detailed simulations are required to make further predictions. In the above example, we have allowed the inner core ( $c$ ) and the overall scale radius of the halo ( $a$ ) to vary freely of each other. However, mergers will certainly correlate the two scales. Also, since halos get built out of smaller halos, the excess mass function analysis should be applied to the smaller building blocks before merger.

Hierarchically-formed dark matter halos from CDM particles without an appreciable phase-space maximum are known to have power-law phase-space profiles,  $Q(r) = \rho(r)/\sigma^3(r) = Q_{-2}(r/r_{-2})^{-\alpha}$ , with  $\alpha \simeq 1.875$  [29]. Here we have normalized the  $Q$  profile at the radius where the log-slope in the density profile is  $-2$ . We can use this result also to estimate the size of the core radius,  $r_{\text{core}}$ , that would be imposed for a given central phase-space density. A minimum estimate for the core size will assume that the power-law continues uninterrupted until the central  $Q$  is reached,  $Q(r = r_{\text{core}}) = Q_0$ . We must first determine the normalization constant  $Q_{-2}$ . We do so assuming an NFW profile [30] and find  $Q(r) \simeq 10^{-11} M_{11}^{-1} (c_{\text{vir}}/15)^{-0.125} (r/R_v)^{-1.875}$ , where  $M_{11} \equiv (M_v/10^{11} h^{-1} M_\odot)$  characterizes the halo virial mass,  $R_v \simeq 134$  kpc  $M_{11}^{1/3}$  is the halo virial radius, and  $c_{\text{vir}} \equiv R_v/r_{-2} \simeq 15$  is appropriate for low-mass spiral galaxy halos [31]. The quoted power-law dependence is approximate but serves to highlight that the dependence is expected to be weak. If we use  $Q(r_{\text{core}}) = Q_0$  to derive a lower limit on the core size and ignore weak  $c_{\text{vir}}$  dependence we obtain  $r_{\text{core}} \gtrsim 2.4 \times 10^{-3} R_v M_{11}^{-0.53} Q_{-6}^{-0.53}$ , where  $Q_{-6} \equiv Q_0/(10^{-6})$  characterizes the  $Q_0$  dependence. Small spiral galaxy halos ( $M_{11} \simeq 1$ ) with  $Q_0$  set by the Lyman- $\alpha$  forest limit for WDM ( $Q_{-6} \simeq 8 \times 10^3$ ) can have only small cores  $r_{\text{core}} \gtrsim 1$  pc. A  $m$ CDM candidate can avoid the Lyman- $\alpha$  forest bound with  $Q_{-6} \gtrsim 10$  and produce larger, dynamically-important cores,  $r_{\text{core}} \gtrsim 100$  pc. As we have emphasized, this estimate represents a conservative lower limit on the full extent of the core region because we have assumed

a sharp break in the phase-space profile. The WDM N-body simulation results of [32] suggest that the cores are  $\sim 3$  times larger than our estimate would give.

Note also that the core size is expected to take up a larger fraction of the halo as we consider smaller systems. Large, soft cores will render these dwarf-size halos quite prone to disruption upon accretion into larger halos [9]. In  $m$ CDM we thus expect the predicted substructure count to be reduced relative to CDM predictions.

## DISCUSSION

For the models considered, the net effect of the injected relativistic energy during decay may be phrased in terms of the effective number of light neutrinos,  $\Delta N_\nu = 1.8 \times 10^{-2} \sqrt{\tau/\text{yrs}}(p_{cm}/m)$ . For the  $m_{\tilde{G}} = 10$  TeV,  $\tau = 5 \times 10^{12}$  s model described above, we have  $\Delta N_\nu \simeq 0.01$  which would be hard to detect.

Neutrinos and photons produced along with  $m$ CDM will arrive unscattered in the diffuse radiation backgrounds today. The strongest constraint comes from the neutrinos produced directly in two-body decays. The relevant low-energy neutrino flux limit is  $< 1.2 \text{ cm}^{-2} \text{ s}^{-1}$  from Super-Kamiokande for energies 18 – 82 MeV [33]. The above sneutrino model is consistent with this limit and in the sensitivity range of future experiments with reduced energy thresholds [34]. Photons may also be produced in three-body decays and subsequent hadronization into neutral pions. Using a very conservative branching fraction of  $10^{-3}$  into photons [35], we find that the photon flux is consistent with observations [36].

In conclusion, we have shown if dark matter is produced from the late decay of cold relics, it will give rise to large cores in low-mass galaxies *and* alleviate the dwarf satellite problem, without destroying agreement with the observed Lyman- $\alpha$  forest power spectrum. This differs from standard WDM models, which, in order to remain consistent with constraints from the Lyman- $\alpha$  power spectrum, cannot produce sizeable cores in small galaxies. Dynamical studies of nearby galaxies may provide the best means to test the  $m$ CDM scenario and compare it to CDM predictions.

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